

Secondary Mass Changes in Vehicle Design Estimation and Application

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Estimation of Secondary Mass Changes in Vehicle Design

1. Introduction—Vehicle design engineers intuitively know that an unplanned mass increase in a component during vehicle design has a ripple effect throughout the vehicle; subsystems need to be resized to function with this additional mass. This further increases vehicle mass even more than the initial mass increase. The phrase 'mass begets mass' describes this phenomenon. A more encouraging view of this behavior is considering a reduction in the mass of a component enabled by a new technology, which then results in a greater mass savings due to the resizing of other subsystems because of the initial mass reduction. These secondary mass changes can be significant, and it is important to consider them during the mass budgeting process. This is because important vehicle metrics, such as fuel consumption and greenhouse gas assessments, depend upon vehicle mass.

Secondary mass reduction may be modeled using subsystem *mass influence coefficients*—the incremental change in subsystem mass for a unit change in gross vehicle mass. This paper focuses on means to estimate influence coefficients and has several objectives:

- 1. Review prior work on secondary mass change estimation
- 2. Suggest uniform terminology for secondary mass change (Appendix 1)
- 3. Review secondary mass change modeling equations
- 4. Provide and compare estimates for mass influence coefficients using two methods: Analytical and Regression.
- 5. Provide a recommended application of secondary mass change analysis to vehicle design

2. Secondary Mass Reduction Potentials - State-of-the-art—Generally within a detailed literature research, several values for the relationship between secondary and the primary mass change potentials are identified and as many as 1300 different sources are screened. However, only 48 publications deal with the topic "secondary mass changes". Eighteen sources estimate empirical values in a wide range of 0.16 and 1.0 for the secondary mass reduction potential, without conducting scientific analyzes to confirm these statements. Only five publications comprise detailed information about the calculation approach and use equations for the determination of the secondary mass reduction potential. Figure 1 gives an overview of the different values identified within the literature research (first five sources with scientific approach). Here, no statements about possible iterations (simple or compounded secondary mass change) are made.

The scientific approaches are summarized in the following. In the year 2000, Audi AG published a study about the influence of different vehicle parameters on driving performance and fuel consumption. The fuel consumption of a conventional C class vehicle and the Audi A2 (1.41 TDI) were compared. During the development phase of the Audi A2, the engine concept, the gear layout, the vehicle efficiency and the overall weight were optimized in order to reduce the driving resistance parameters and, thus, the fuel consumption of the vehicle. Through the use of an Aluminum Space Frame, the body weight was reduced with primary measures by about 100 kg compared to a conventional car. The optimization of the vehicle equipment and the size revision of the vehicle resulted in a further primary mass reduction of 34 kg, the engine displacement and the cooling system of the vehicle could be resized and light chassis components could be used, while maintaining vehicle driving performance. So, with the primary mass changes of 134 kg overall, the weight of the powertrain could be secondarily reduced by 31 kg and the weight of the



chassis and the fuel tank system could be secondarily reduced by 65 kg. As a conclusion of this, the relationship between the secondary and the primary mass change is about 0.72 [1].

In 2003 the Forschungsgesellschaft Kraftfahrwesen mbH Aachen (fka) and the European Aluminum Association (EAA) analyzed lightweight design potentials of an aluminum-intensive vehicle. In doing so, a reference steel C class (e.g. VW Golf) vehicle was defined by its components. Following, the relevant steel components of the reference vehicle were substituted with aluminum components. Consequently, primary measures reduced the overall weight up to 226 kg. A further secondary mass change of 116 kg was feasible. The result is a relationship between the secondary and primary mass reduction of about 0.51 [2]. These results were published in a 2004 study focused on increasing aluminum use in vehicles [3].



Figure 1 Primary to secondary mass reduction potentials

In 2008 the Novelis Inc. and IBIS Associates Inc. analyzed the usage of aluminum vehicle structures in combination with alternative powertrain technologies. The study focused on aluminum usage in the vehicle body, powertrain and chassis. Starting with a defined steel reference C class vehicle, the overall weight of the vehicle was estimated under the following scenarios:

- Steel body with a conventional internal combustion engine (reference)
- Aluminum body with a conventional internal combustion engine
- Steel body with a hybrid powertrain
- Aluminum body with a hybrid powertrain
- Steel body with a diesel engine (direct fuel injection)
- Aluminum body with a diesel engine (direct fuel injection)

In each scenario, the conventional steel components of the reference car were substituted by aluminum components (primary mass change). The engine, 12 volt battery, exhaust system, fuel tank, gearbox, subframe, chassis, braking system, tires, steering system and bumpers were



analyzed for possible secondary mass changes. The study results indicated that a primary mass change of 159 kg caused a secondary mass change of 101 kg, independent of the powertrain systems of the various scenarios. As a conclusion of this, the relationship between the secondary and the primary mass reduction is about 0.64 (independent of the powertrain systems) [4].

In 1999 Professor H.H. Braess published the first analytical approach for the secondary mass change calculation. In Braess' approach, the overall vehicle weight includes the powertrain, chassis, body, body equipment and fuel. The approach, in which several equations for the calculation of the secondary mass change were used, leads to the assumption, that the 100 kg increase in vehicle weight (e.g. caused by safety standards, increasing comfort demands of the customer, etc.) correlates to a further 16 kg weight increase of the vehicle components [5].

3. Secondary Mass Change Modeling — To formalize models of secondary mass change, it is necessary to precisely describe the assumed vehicle design context. We begin having a balanced vehicle design—that is all mass dependent subsystems are sized to the gross vehicle mass, M_0 , for the particular vehicle. We further assume that the vehicle is yet in the design stage, and we are free to redesign the subsystems. These assumptions define the *initial* vehicle and subsystems.

Now an unplanned mass change, Δ , occurs in a component. This change upsets the balanced design, and the subsystems are no longer sized for the current vehicle mass $(M_0+\Delta)$. We now assume that the subsystems are redesigned to resize them for this new vehicle mass. This subsystem resizing will cause additional—secondary—changes in the subsystem mass. These changes may be added to result in the vehicle's secondary mass change.

This resizing may be done as a series of design iterations. If one iteration is completed, the secondary mass change is referred to as *simple secondary mass change*. However, note that after the first redesign, the vehicle subsystems are sized for a vehicle of mass $(M_0+\Delta)$. Yet the mass of the vehicle is now $(M_0+\Delta + simple secondary mass change)$. Thus the vehicle subsystems are not yet in balance and may be redesigned again. We can imagine that this iterative redesigning occurs theoretically over an infinite number of cycles. The mass changes eventually converges to a larger secondary mass change—the *compounded secondary mass change* (an example of this iterative resizing is provided later).

This behavior is captured in the following equations. The overall vehicle mass after resizing, M_{RS} , is

$$M_{RS} = M_0 + \Delta + \Delta \Gamma \qquad \text{equation 1}$$

Where

 $\begin{array}{ll} M_0 & = \mbox{Initial vehicle mass for which the subsystems are sized} \\ \Delta & = \mbox{Initial total mass change } (primary mass change) \\ M_{RS} & = \mbox{Vehicle mass after resizing subsystems} \\ \Delta^{\bullet}\Gamma & = \mbox{Additional } (secondary) \mbox{ mass change due to resizing subsystems} \\ \Gamma & = \mbox{Secondary mass coefficient which depends on the treatment of resizing iterations.} \\ Simple for one resizing \\ Compounded for an infinite series of resizings \\ \end{array}$

The secondary mass coefficient, Γ , depends on subsystem mass influence coefficients, γi —the change in mass of subsystem i when the gross vehicle mass increases by l kg.



The simple secondary mass coefficient assuming one resizing iteration is

$$\Gamma = \gamma_V$$
 equation 2

Assuming that the subsystem mass influence coefficients are constant, and that the sum is less than one, $0 < \gamma_V < 1$, then after an infinite number of resizing iterations we have the secondary mass coefficient for compounded secondary mass change

$$\Gamma = \frac{\gamma_V}{1 - \gamma_V}$$
 equation 3

The mass influence coefficient for the vehicle, γ_V , is given by

$$\gamma_V = \Sigma \gamma_i$$
 equation 4

where γi is the mass influence coefficient for subsystem *i*.

The resulting mass for subsystem i due to an initial vehicle mass increase of Δ is

$$mi_{RS} = mi_0 + \Delta_i + \Delta \tau$$
 equation 5

$$\tau = \gamma i$$
 for simple secondary mass change equation 6
 $\tau = \frac{\gamma i}{1 - \gamma_V}$ for compounded secondary mass change equation 7

where

*mi*₀ =Initial subsystem i mass

 Δ_i =Initial mass change in subsystem i

 mi_{RS} =Resized subsystem i mass

 $\Delta \cdot \tau$ =Additional (secondary) mass change for subsystem i

To illustrate the use of these equations, consider a vehicle with the subsystem influence coefficients shown in Figure 2, second column from left (using influence coefficients from the analytical approach of fka shown in Section 5 [26]). A primary mass reduction of $\Delta = -100 \text{ kg}$ occurs. In the first design iteration, each subsystem mass is reduced by the product of the primary mass reduction and the subsystem influence coefficient.



Figure 2 Illustration of secondary mass change calculation (Using influence coefficient data from [26])

These mass reductions are shown in the column labeled *Resizing iteration number 1*. The sum of these mass reductions is the *simple secondary mass change* for the vehicle: -34.15 kg. However, the vehicle subsystems are not yet balanced as this -34.15 kg change has not yet been taken into account. Thus in iteration 2, subsystem mass may be further reduced as shown. This process may be repeated. Each iteration will result in a smaller secondary mass reduction. Four iterations are shown in Figure 2 to illustrate this compounding behavior.

For an infinite number of iterations, the cumulative mass change for the vehicle, $\Delta\Gamma$, converges to a value given by equation 3. Whether *simple* or *compounded* secondary mass change is most appropriate will depend on the particular design context. If the design activity is at the product planning stage, then the full advantage of compounded secondary mass change may be gained. If the design activity is at the detail design stage, simple secondary mass change would be a more appropriate reflection of the constraints on the ability to redesign subsystems.

4. Means to Estimate Influence Coefficients—It is clear from equations 1 to 7 that estimations of secondary mass change depend on the subsystem influence coefficients, γ_i for the vehicle mass dependent subsystems. Three methods to estimate mass influence coefficients for these subsystems are Ratio, Regression, and Analytic, Figure 3.

In the Ratio method, a linear relationship is assumed between subsystem mass and gross vehicle mass—GVM. A representative reference vehicle is selected and the ratio between subsystem



mass and gross vehicle mass is determined for each mass dependent subsystems and becomes the influence coefficient. The appeal of the ratio method is its modest data requirements (mass data from only one vehicle is sufficient for estimation). However, its assumption of linearity and the assumed sole dependence of subsystem mass on gross vehicle mass results in a consistent bias on the high side. Therefore the ratio method is typically used only for a quick check on estimates from other methods.



Figure 3 Methods to estimate secondary mass influence coefficients

For more accurate estimates of mass influence coefficients, both the Analytical method and Regression method are used in practice today to a limited degree. It is the intent of this paper to extend and improve both secondary mass change data and methodology. In the following sections, recent work is described in determining mass influence coefficients estimates for sedans and hatchbacks having integral body structure, front wheel drive, and internal combustion engine. Work on the analytical and regression methods was completed independently with slightly different subsystem definitions used, as outlined in each of the following sections.

5. Analytical Method—The analytical method contains the development of a classification system for the analytical determination of the simple and compounded secondary mass reduction in vehicles. Based on defined selection criteria, all components the vehicle body, powertrain, chassis (consisting of the subsystems suspension, braking system, steering system and tires and rims), electronics and interior with secondary mass reduction potential are identified. Based on this, empirical and analytical relationships between component properties and masses are developed for the calculation of the secondary mass reduction potential. The analytical approach is presented in the paragraphs following.

In order to calculate vehicle secondary mass reduction potential, all components that can



contribute to the mass reduction must be identified. In doing so, selection criteria have to be defined. In the methodology described following, it is assumed that components whose dimensioning depends on the gross vehicle mass, driving power, driving torque, inertia forces, fuel consumption and energy absorption, have potential for secondary mass reduction. Based on this it can be considered that the vehicle body, powertrain and chassis (consisting of the subsystems suspension, braking system, steering system and tires and rims) have secondary mass reduction potential. The vehicle electronics and interior show no secondary mass reduction potential. Considering a C class vehicle with a curb mass of 1405 kg (e.g. VW Golf), it can be assumed that about 62 % (868 kg) of the vehicle mass can be affected by secondary mass reduction.

The secondary mass reduction potential of the **body structure** (including bumpers) is analyzed by crash simulations. So, using a VW Golf-based reference model, the following load cases are analyzed:

- Euro NCAP frontal impact (deformable barrier, 40 % offset, 64 km/h)
- Euro NCAP side impact (mobile deformable barrier, 50 km/h)
- FMVSS 301 (rear impact with rigid barrier, full width, 48 km/h)

Body components with the highest energy absorption are defined for the determination of the secondary mass reduction. With the assumption of a primary mass change of 100 kg, the sheet material thicknesses of the relevant body components are reduced until reaching the same crash performance between the reference and 100 kg primary mass-reduced vehicle, in terms of constant component intrusions.

With regard to the Euro NCAP front impact analysis, 20 % of the kinetic energy is absorbed by the front bumper system and the longitudinal beams. By decreasing the sheet material thicknesses of these components, but maintaining intrusion performance, the overall weight can be reduced from 19.97 kg to 12.82 kg. In the side impact test, 26 % of the kinetic energy is absorbed by the B-pillar, floor, the seat cross members, sill and side panel components. With the assumption of a primary mass change of 100 kg, the component weights can be decreased from 58.2 kg to 57.26 kg. In the FMVSS 301 rear crash test, the component weights of the rear longitudinal beams, rear bumper system and rear floor can be decreased from 28.74 kg to 26.95 kg. The result in this example, when considering a **100 kg** primary mass change in the 1405 kg vehicle curb mass, is a secondary mass reduction of the **body structure** by **9.61 kg** [26].

In order to determine the secondary mass change of the **powertrain** (engine, gearbox, differential, clutch system, cooling system, 12 volt battery, drive and cardan shafts) it is assumed, that the vehicle with 100 kg primary mass change and the reference vehicle share about the same driving performance in terms of constant vehicle accelerations. In the following example, it is defined that the available excess power in each gear should be consigned completely to vehicle acceleration. The driving resistance power of the vehicle can be determined by equation 8. (Nomenclature for formula symbols and indices for this section is found in Appendix 3.)

$$P_{Bed} = (F_R + F_L + F_B + F_{St}) \cdot v_{Fzg} = F_{Bed} \cdot v_{Fzg}$$
equation 8

Using data on the speed-torque characteristics of the engine, gearbox ratio (i_n) and the differential ratio (i_{Diff}) , the following relation to engine speed, driving torque and vehicle speed can be defined:



$$\mathbf{n}_{\text{Antr.}} = \frac{\mathbf{v}_{\text{Fzg}} \cdot \mathbf{i}_{n} \cdot \mathbf{i}_{\text{Diff}}}{2\pi \cdot \mathbf{r}_{\text{dyn}}} = \frac{\mathbf{P}_{\text{Antr.}}}{2\pi \cdot \mathbf{M}_{\text{Antr.}}}$$
equation 9

By the use of equation 8 and 9, the new engine map for the vehicle with primary mass change can be determined. In Figure 4 and in the powertrain discussion following, the reduced engine torque of the primary mass-changed vehicle is used for the dimensioning of the different components of the powertrain.



and the vehicle with primary mass change (-100 kg) [26]

The first step for the dimensioning of the manual 5 or 6 speed gearbox consists of calculating the distance of the shaft center between the gearbox input shaft and the gearbox output shaft (equation 10). In doing so, the ratio of the first gear and the constant gearbox layout factors Z_i and K_i are used. The gear wheel ratios of the gears 2 to 5 or 2 to 6 are determined depending on a defined gearbox ratio spread $i_{G,ges}$ by the use of the progressive ratio φ_1 and the factor φ_2 . Based on the calculated ratios, the gear wheel diameters ($d_{z,n}$) and the pinion diameters ($d_{r,n}$) can be determined (equation 13 and 14).



$$a_{Get} = \sqrt[3]{\frac{M_{Antr.} \cdot (i_1+1)^4}{4 \cdot i_1 \cdot \frac{b}{d_1}}} \cdot \sqrt[3]{\frac{(Z_{B/D}Z_H Z_E Z_E Z_\beta S_H)^2 \cdot (K_A K_V K_{H\beta} K_{H\alpha})}{(\sigma_{H.lim} Z_{NT} Z_L Z_R Z_V Z_W Z_X)^2}} equation 10$$

$$\phi_1 = \sqrt[z-1]{\frac{i_{G,ges}}{\phi_2^{0.5 \cdot (z-1) \cdot (z-2)}}} equation 11$$

$$i_n = i_z \cdot \phi_1^{(z-n)} \cdot \phi_2^{0.5 \cdot (z-n) \cdot (z-n-1)} equation 12$$

$$d_{r,n} = \frac{2 \cdot a_{Get}}{1+i_n} equation 13$$

$$d_{z,n} = (2 \cdot a_{Get}) - d_{r,n} equation 14$$

The length of the gearbox can be determined by the summation of the component width of the gear pinions $b_{r,i}$, bearings b_L , detents b_{SK} and by the number of gears (z+1) (equation 15). In this case, the factors A_L and B_S describe the number of bearings and detents in the gearbox. Beside the gearbox length, the gearbox diameter can be determined (equation 16). For the gearbox housing, a hollow cylinder with a sheet material thickness of 11 mm is defined. After the calculation of the component dimensions, the weights of the gear box components (gear wheels, gear shafts, etc.) can be calculated by the multiplication of the component volume and the component density. The weight of the vehicle differential can be calculated in the same manner.

$$l_{Get} = ((z+1) \cdot b_{r,i}) + (A_L \cdot b_L) + (B_S \cdot b_{Sk})$$
 equation 15

$$d_{Get} = a_{Get} + 0.5 \cdot d_{r,1} + 0.5 \cdot d_{z,1}$$
 equation 16

$$G_{r,i} = (d_{r,i}^{2} - d_{GEW,min}^{2}) \cdot \frac{\pi}{4} \cdot b_{r,i} \cdot \rho_{Stahl}$$
equation 17

For the dimensioning of the drive shafts and the cardan shaft (if available), the drive shafts and the joints are considered separately. The required diameter of the drive shafts can be calculated by taking into account the ratios of the first gear, the differential (i_1 and i_{Diff}) and the engine torque $M_{Antr.}$, as well as the torsion fatigue strength $\tau_{b,W}$ and the safety index S_{AW} and S_{KW} . It is considered, that the drive shafts are solid and the cardan shaft is a hollow cylinder with a sheet material thickness t_{KW} of nearly 3 mm.

$$d_{AW} \ge \sqrt[3]{\frac{16 \cdot S_{AW} \cdot M_{Antr.} \cdot i_{1} \cdot i_{Diff}}{\pi \cdot \tau_{b,W}}}$$
equation 18
$$t_{KW} \cdot d_{KW} \ge \frac{2 \cdot S_{KW} \cdot M_{Antr.} \cdot i_{1}}{\pi \cdot \tau_{b,W}}$$
equation 19

After defining the drive shaft length, which depends on the vehicle dimensions, the component weights can be calculated by multiplication of the component volume and the component density.



Based on existing vehicle data, 1.5 kg of total weight for the drive shafts joints (AW) and 4kg of total weight for cardan shaft joints (KW) should be added to equations 20 and 21.

$$G_{AW} = l_{AW} \cdot \pi \cdot \left(\frac{d_{AW}}{2}\right)^2 \cdot \rho_{Stahl}$$
equation 20
$$G_{KW} = l_{KW} \cdot \pi \cdot \left(\left(\frac{d_{KW}}{2}\right)^2 - \left(\frac{d_{KW}}{2} - t_{KW}\right)\right) \cdot \rho_{Stahl}$$
equation 21

To calculate the engine dimensions, an empirical relationship between the engine torque and weight (relative to the driving torque) is used. The engine subsystem includes the fuel injection, engine block, engine mounting, cylinder head, crank shaft, belt drive, oil supply, cooling system and turbocharging (for diesel engines and turbocharged gasoline engines), In this example, approximately 100 existing vehicle applications are analyzed and transferred into the empirical equation.

$$G_{AEW} = 0.2998 \cdot M_{Antr.} + 59.24 \qquad \text{equation } 22$$

In addition to the approach for the engine weight calculation, the individual component weights of the engine speed converter, consisting of the flywheel (SR), clutch pressure plate (KDP) and clutch disc (KS), is calculated by an empirical relationship between the component weights and the engine torque. So, approximately 100 existing vehicle applications are analyzed and transferred into the empirical equations (equation 23 to 25).

$$\begin{split} G_{DZW,SR} &= 0.0309 \cdot M_{Antr.} + 3.5157 & equation 23 \\ G_{DZW,KS} &= 0.001 \cdot M_{Antr.} + 0.792 & equation 24 \\ G_{DZW,KDP} &= 0.0098 \cdot M_{Antr.} + 1.8785 & equation 25 \end{split}$$

For the calculation of the additional energy storage component weights G_{ZES} [12 volt battery for gasoline engines (OM) and diesel engines (DM)] and the cooling system G_K , consisting of the cooler, cooling hoses, fan and fan motor, the fluids G_F [cooling water (KW) and engine oil (MÖ)] empirical relationships between the component weights and the engine torque specifications from approximately 100 existing vehicles are used (equations 26 to 30) [26].

$$G_{ZES,OM} = 0.1298 \cdot G_{AEW} + 1.2152$$
 equation 26

$$G_{ZES,DM} = 0.0935 \cdot G_{AEW} + 4.872 \qquad \text{equation } 27$$

$$G_{K} = 0.0177 \cdot M_{Antr.} + 2.6902$$
 equation 28

$$G_{F,KW} = 0.0171 \cdot M_{Antr.} + 2.2155$$
 equation 29

$$G_{F,MO} = 0.0092 \cdot M_{Antr.} + 1.8953$$
 equation 30

Secondary weight reduction of the **powertrain** of **10.63 kg**, can be achieved, when considering a **100 kg** primary mass change [26].



In order to calculate the **fuel tank system** dimensions, knowledge of the vehicle range is needed, which in turn requires an estimation of the engine fuel consumption. To determine the fuel consumption of a specific engine, the Willans methodology [30] can be used, which assumes that the vehicle fuel consumption can be divided into zero-power consumption V_{Null} and effective power V_{Pe} (equation 31).

$$V = V_{Null} + V_{P_e}$$
 equation 31

The zero-power consumption can be calculated using equation 32. The coefficients a_V , b_V and c_V are defined to 0.076, 0.17 and 0.2 (for gasoline engines) and 0.08, 0.075 and 0.1 (for diesel engines). V_H represents the engine capacity and v_{Fzg} the vehicle speed.

$$\mathbf{V}_{\text{Null}} = \left(\mathbf{a}_{\text{V}} \cdot \frac{\mathbf{v}_{\text{Fzg}}^2}{\mathbf{v}_{1000}^2} + \mathbf{b}_{\text{V}} \cdot \frac{\mathbf{v}_{\text{Fzg}}}{\mathbf{v}_{1000}} + \mathbf{c}_{\text{V}}\right) \cdot \mathbf{V}_{\text{H}}$$
equation 32

The share of effective power can be calculated using equation 33. For the constant coefficient z_{Pe} , a value of 0.264 l/kWh can be assumed for gasoline engines and 0.208 l/kWh for diesel engines.

$$V_{P_e} = z_{P_e} \cdot P_e$$
 equation 33

Given that the fuel tank volume $V_{KT,Ref.}$ is known, the range of the reference vehicle can be calculated via the fuel consumption V_{Ref} and the speed difference of the NEDC Δv_{NEDC} as shown in equation 34. The fuel tank volume of the weight-reduced vehicle results according to equation 35. The weight of the fuel tank can be calculated with equation 36.

$$S_{\text{Ref.}} = \frac{V_{\text{KT,Ref.}}}{V_{\text{Ref.}}} \cdot \Delta v_{\text{NEDC}}$$
equation 34
$$V_{\text{KT,gew.red.}} = \frac{V_{\text{gew.red.}} \cdot S_{\text{Ref.}}}{\Delta v_{\text{NEDC}}}$$
equation 35

$$G_{\rm KT} = 2.1942 \cdot e^{0.0227 \cdot V_{\rm KT}}$$
 equation 36

Using this calculation methodology, an analysis published a **1.01 kg** secondary weight reduction in the **fuel tank system**, when considering a **100 kg** primary mass change [26].

The dimensioning of the **suspension** and the suspension components depends largely on the amount of the gross vehicle mass (GVM). Therefore, the GVM is defined as one of the most important input parameters for the layout of the chassis components.

In order to resize the components of the lateral dynamics, which are responsible for the wheel guidance, several FE-simulations are conducted. The McPherson-front axle and the control blade suspension rear axle are treated separately. On the basis of the simulation results of the reference vehicle, all loads and stiffnesses of the front and the rear axle individual components are reduced by 10 %. Subsequently, an adjustment of the sheet material thicknesses can be made to the lighter vehicle, while maintaining the reference vehicle component stresses.

To identify secondary mass reduction potential of the vertical dynamics subsystem, which comprises the spring and damper (separated into front and rear axle), empirical relationships are drawn between the component weights and the gross vehicle mass by comparing the specifications of approximately 100 existing vehicles (equations 37 and 38). These equations are valid for McPherson-front axles and multi-link rear axles.



$$G_{VD,VA} = 0.0031 \cdot G_{Fzg,zul.} + 1.7258$$
 equation 37
 $G_{VD,HA} = 0.0045 \cdot G_{Fzg,zul.} - 3.9097$ equation 38

Citing the same analysis mentioned previously, a 4.95 kg secondary weight reduction in the suspension was noted, when considering a 100 kg primary mass change [26].

To calculate the **braking system dimensions**, the braking distance, based on a speed of 100 km/h, must be defined. By using this approach, the braking deceleration can be calculated and should be used for both the reference and the primary mass-changed vehicle (equation 39).

$$a_{\rm Br} = 0.5 \cdot \frac{\Delta v_{\rm Fzg}^2}{s_{100\rm km/h}}$$
 equation 39

In a next step, the wheel loads of the front $(F_{R,Z,v})$ and rear axle $(F_{R,Z,h})$ have to be determined by the use of the mean deceleration and the vehicle center of gravity. In this context, I designates the wheel base, h the height of the center of gravity, with l_h and l_v being the distance between the center of gravity and the front and rear axles, respectively (equations 40 and 41).

$$F_{R,Z,v} = \frac{G_{Fzg,zul.} \cdot g \cdot \frac{l_{h}}{l} + G_{Fzg,zul.} \cdot a_{Br} \cdot \frac{h}{l}}{2}$$
equation 40
$$F_{R,Z,h} = \frac{G_{Fzg,zul.} \cdot g \cdot \frac{l_{v}}{l} + G_{Fzg,zul.} \cdot a_{Br} \cdot \frac{h}{l}}{2}$$
equation 41

Taking the coefficient of static friction of the street surface μ_{St} into account, the maximum braking power at the front $F_{Br,v}$ and at the rear wheel $F_{Br,h}$ can be calculated as follows:

2

$$F_{Br,v} = \mu_{Str.} \cdot F_{R,Z,v}$$
 equation 42

$$F_{Br,h} = \mu_{Str.} \cdot F_{R,Z,h}$$
 equation 43

By defining an effective diameter of the front $D_{Br,S,v}$ and rear $D_{Br,S,h}$ break disc, as well as the reference vehicle dynamic wheel radius r_{dyn}, the braking power at the effective diameter in the front $F_{Br,S,v}$ and in the rear $F_{Br,S,v}$ can be calculated as follows:

| $\mathbf{F}_{\mathbf{Br},\mathbf{S},\mathbf{v}} = \mathbf{F}_{\mathbf{Br},\mathbf{v}} \cdot$ | $\frac{2 \cdot r_{dyn}}{D_{Br,S,v}}$ | equation 44 |
|--|--------------------------------------|-------------|
| $\mathbf{F}_{\mathrm{Br},\mathrm{S},\mathrm{h}} = \mathbf{F}_{\mathrm{Br},\mathrm{h}} \cdot$ | $\frac{2 \cdot r_{dyn}}{D_{Br,S,h}}$ | equation 45 |

In the following equations, a constant effective diameter of the braking disc (front and rear) for the reference and the primary mass-changed vehicle is defined. At a constant, dynamic wheel radius r_{dvn}, the effective diameter of the braking disc in the front D_{Br,S,v,gew.red.} and in the rear D_{Br,S,h,gew.red.} is calculated at constant braking acceleration a_{Br} and adapted wheel loads F_{Br,v,gew,red.} and F_{Br,h,gew,red} according to equations 46 and 47.



$$D_{Br,S,v,gew.red.} = F_{Br,v,gew.red.} \cdot \frac{2 \cdot r_{dyn}}{F_{Br,S,v}}$$
equation 46
$$D_{Br,S,h,gew.red.} = F_{Br,h,gew.red.} \cdot \frac{2 \cdot r_{dyn}}{F_{Br,S,h}}$$
equation 47

Based on these assumptions, the component weights can be calculated using the equations 48 to 51.

$$\begin{split} G_{FW,Br,S,innenbel,,v} &= 0.5443 \cdot e^{0.0089 \cdot D_{Br,S,außen,v,i}} & \text{equation 48} \\ G_{FW,Br,S,massiv,v} &= 0.0554 \cdot D_{Br,S,außen,v,i} - 9.7911 & \text{equation 49} \\ G_{FW,Br,S,innenbel,,h} &= 0.0008 \cdot D_{Br,S,außen,h,i}^{1.5745} & \text{equation 50} \\ G_{FW,Br,S,massiv,h} &= 0.0238 \cdot D_{Br,S,außen,h,i} - 2.0143 & \text{equation 51} \end{split}$$

Equation 52 shows the empirical relationship for the calculation of the steering system component weight.

$$G_{LS} = 0.006 \cdot G_{Fzg,zul} + 10.923$$
 equation 52

Using the referenced calculation, a **0.70 kg** secondary mass reduction is achieved in the **steering system**, when considering a **100 kg** primary mass change [26].

Determining the dimensions of the **tires and rims** depends primarily on the gross vehicle mass, the maximum vehicle velocity and the braking system dimensions. In accordance with DIN 7803, only 50, 55, 60 and 65 series steel rims and tires are referenced. In this context, the tire series indicates the dimensions of the tire wall (e.g., 50 series indicates, that the tire wall dimension is 50 % of the tire tread). Depending on the gross vehicle mass and with the knowledge of the axle load distribution the maximum static wheel loads (front $F_{Z,W,v}$ and rear $F_{Z,W,h}$) can be determined as follows:

$$F_{Z,W,v} = \frac{1}{2} \cdot G_{Fzg.,zul.} \cdot \frac{l_h}{l}$$
equation 53
$$F_{Z,W,h} = \frac{1}{2} \cdot G_{Fzg.,zul.} \cdot \frac{l_v}{l}$$
equation 54

By the use of a safety index of $S_R = 0.9$, the required tire load can be calculated (equation 55).

$$G_{R,Z} = F_{Z,W,max} \cdot S_R$$
 equation 55

$$F_{Z,W,max} = Max \{F_{Z,W,v} | F_{Z,W,h}\}$$
 equation 56



The required tire load $G_{R,Z}$ can be converted into load indices (LI), according to DIN 7803 as follows:

| LI | G _{R,Z} [kg] | LI | G _{R,Z} [kg] | LI | G _{R,Z} [kg] | LI | G _{R,Z} [kg] | LI | G _{R,Z} [kg] |
|----|-----------------------|----|------------------------------|----|-----------------------|----|-----------------------|-----|-----------------------|
| 50 | 190 | 62 | 265 | 74 | 375 | 86 | 530 | 98 | 750 |
| 51 | 195 | 63 | 272 | 75 | 387 | 87 | 545 | 99 | 775 |
| 52 | 200 | 64 | 280 | 76 | 400 | 88 | 560 | 100 | 800 |
| 53 | 206 | 65 | 290 | 77 | 412 | 89 | 580 | 101 | 825 |
| 54 | 212 | 66 | 300 | 78 | 425 | 90 | 600 | 102 | 850 |
| 55 | 218 | 67 | 307 | 79 | 437 | 91 | 615 | 103 | 875 |
| 56 | 224 | 68 | 315 | 80 | 450 | 92 | 630 | 104 | 900 |
| 57 | 230 | 69 | 325 | 81 | 462 | 93 | 650 | 105 | 925 |
| 58 | 236 | 70 | 335 | 82 | 475 | 94 | 670 | 106 | 950 |
| 59 | 243 | 71 | 345 | 83 | 487 | 95 | 690 | 107 | 975 |
| 60 | 250 | 72 | 355 | 84 | 500 | 96 | 710 | 108 | 1000 |
| 61 | 257 | 73 | 365 | 85 | 515 | 97 | 730 | 109 | 1030 |

Figure 5

Relationship between tire load $(G_{R,Z})$ and load index (LI) [26]

The choice of a usable tire for a specific vehicle depends on the calculated load index (LI), the required rim diameter, which depends on the dimensions of the braking system, and the tire series. So for example, Figure 6 shows an extract of DIN 7803 for radial tires of the 55 series, up to a maximum vehicle velocity of 160 km/h. For a load index of 88 and a required 15-inch rim diameter, a 205/55 R15 tire would be usable.

The tire weight depends on the tire width and the tire diameter. Consequently, empirical relationships between the tire widths, diameters and weights for 200 existing tires are defined according to tire series.

| Radial tire series 55 (up to 160 km/h) | | | | | | |
|--|------------|------------|---------------------|-----------------------|----------------|--|
| LI | Tire size | Tire width | Rim diameter [inch] | Rim base width [inch] | Rim width [mm] | |
| 68 | 165/55 R12 | 165 | 12 | 5.0 B | 127 | |
| 70 | 165/55 R13 | 165 | 13 | 5.0 B | 127 | |
| 72 | 165/55 R14 | 165 | 14 | 5.0 J | 127 | |
| 77 | 175/55 R15 | 175 | 15 | 5.5 J | 139.7 | |
| 77 | 185/55 R13 | 185 | 13 | 6.0 B | 152.4 | |
| 80 | 185/55 R14 | 185 | 14 | 6.0 J | 152.4 | |
| 82 | 185/55 R15 | 185 | 15 | 6.0 J | 152.4 | |
| 80 | 195/55 R13 | 195 | 13 | 6.0 B | 152.4 | |
| 82 | 195/55 R14 | 195 | 14 | 6.0 J | 152.4 | |
| 85 | 195/55 R15 | 195 | 15 | 6.0 J | 152.4 | |
| 85 | 205/55 R14 | 205 | 14 | 6.5 J | 165.1 | |
| 88 | 205/55 R15 | 205 | 15 | 6.5 J | 165.1 | |
| 89 | 205/55 R16 | 205 | 16 | 6.5 J | 165.1 | |

Figure 6

Choice of a usable tire for a specific vehicle (according to DIN 7803)

For radial tires of 50 series (16 inch diameter), 55 series (17 inch), 60 series (18 inch) and 65 series (15 inch) the following relationships between the tire width b_R and the tire weight $G_{b,Serie}$ can be defined:



| equation 57 | $G_{b,Serie50,16''} = 0.0581 \cdot b_R - 2.8798$ |
|-------------|--|
| equation 58 | $G_{b,Serie55,17''} = 0.0543 \cdot b_R - 1.7488$ |
| equation 59 | $G_{b,Serie60,18''} = 0.0586 \cdot b_R - 0.387$ |
| equation 60 | $G_{b,Serie65,15''} = 0.0549 \cdot b_R - 0.4247$ |

Based on the relationship between the tire width and weight for a constant tire diameter, the following relationship between the diameter and weight G_{FW} can be defined for the tire series:

$$G_{FW,Reifen,Serie50} = G_{b,Serie50,16"} \cdot \frac{A_{Flanke,R,Serie50}}{A_{Flanke,R,Serie50,16"}}$$
equation 61

$$G_{FW,Reifen,Serie55} = G_{b,Serie55,17"} \cdot \frac{A_{Flanke,R,Serie55}}{A_{Flanke,R,Serie55,17"}}$$
equation 62

$$G_{FW,,Reifen,Serie60} = G_{b,Serie60,18"} \cdot \frac{A_{Flanke,R,Serie60}}{A_{Flanke,R,Serie60,18"}}$$
equation 63

$$G_{FW,Reifen,Serie65} = G_{b,Serie65,15"} \cdot \frac{A_{Flanke,R,Serie65}}{A_{Flanke,R,Serie65,15"}}$$
equation 64

The area of the tire wall can be calculated according to equation 65.

$$A_{\text{Flanke},\text{R},\text{i}} = \pi \cdot \left[\left(\frac{0.02 \cdot k_{\text{L/F}} \cdot b_{\text{R}} + 25.4 \cdot D_{\text{Felge}}}{2} \right)^2 - \left(\frac{25.4 \cdot D_{\text{Felge}}}{2} \right)^2 \right] \quad \text{equation } 65$$

For the calculation of the rim weight can be done by the knowledge of the rim sheet thickness t_{Felge} , rim diameter D_{Felge} , rim width $B_{M,F}$ and material density ρ_{Stahl} [26].

$$G_{FW,F} = \pi \cdot \frac{7.5}{100^3} \cdot \rho_{Stahl} \cdot t_{Felge} \cdot \left[\left(\frac{25.4 \cdot D_{Felge}}{2} \right)^2 + \frac{25.4 \cdot D_{Felge}}{2} \cdot B_{M,F} \right] \quad \text{equation 66}$$

According to this calculation, a **3.58 kg** secondary weight reduction can be achieved in the **tires** and **rims**, when considering a **100 kg** primary mass change [26].

The simple secondary mass change of the whole vehicle amounts to **34.15 kg**, if when considering a **100 kg** primary mass change. If multiple iterations are conducted, the compounded secondary mass change reaches a maximum of **51.86 kg** [26].

6. Regression Method—In the regression method, a statistical model is fit to empirical subsystem mass data for a set of vehicles, center graph of Figure 3. In this method we are assuming that the vehicles have similar performance requirements, that subsystem technology is similar, and that subsystems for each vehicle are sized to the particular GVM of that vehicle. Clearly these assumptions are violated for several real conditions. First, when a vehicle shares the body platform as a member of a common architecture, the platform is sized to the heaviest member—not the particular vehicle under study. Second, for a model offering a range of powertrain options, subsystems are sized to the heaviest powertrain, which may be different than for the particular vehicle. Additionally, not all vehicles in the database will have the same levels of performance, and subsystem mass differences will appear for vehicles having the same gross



vehicle mass because of these performance differences. These violations of the assumptions will result in inflating the residual error in the statistical models. Thus, while we have not avoided these assumption violations, we have quantified them with error bands.

Two general models may be used to predict subsystem mass: Linear additive model and Power model. Both include the dependence of subsystem mass on mass drivers including GVM. For example, gross vehicle mass is a mass driver for the body structure. This is because GVM influences structure requirements for front barrier impact, for magnitude of loads through the suspension, for roof crush, and thereby influences body mass. Another mass driver for the body is physical size, expressed as projected plan view area. Larger areas will require more material to enclose the interior space even for an identical GVM. To determine significant mass drivers, engineering judgment is first used to identify a set of potential mass drivers for each subsystem, and then statistical significance is used to determine which to include in the model. The form of each model is shown below, equation 67 and equation 69. In these equations subsystem mass, \hat{m}_i , is estimated with known mass driver values where β_{-} are coefficients estimated by regression. (Nomenclature for formula symbols and indices for this section is found in Appendix 4.)

Linear model

$$\hat{m}_i = \beta_0 + \beta_1 (massdriver)_1 + \beta_2 (massdriver)_2 + \dots + \varepsilon$$
 equation 67

Taking gross vehicle mass as mass driver 1, the estimate for the subsystem influence coefficient, γ_i , is then

$$\gamma_i = \beta_1$$
 equation 68

Power model

$$\hat{m}_i = \beta_0 (massdriver)^{\beta_1} (massdriver)^{\beta_2} (...) r_{\varepsilon}$$
 equation 69

Taking gross vehicle mass as mass driver 1, the subsystem influence coefficient is

$$\gamma_i = \frac{\partial \hat{m}_i}{\partial (GVM)} \Big|_{\overline{m}_i}$$
 equation 70

As an example of evaluating these models, vehicle mass data for 69 sedan and hatchback vehicles was acquired from a benchmarking database [24]. The vehicles ranged from A class (3400 mm length, 920 kg curb mass) to D class (4800 mm length, 1600 kg curb mass) over the model years 2000 to 2008. All vehicles had similar subsystem technology: Integral steel body, transverse front wheel drive, internal combustion engine, McPherson strut front suspension.

In past applications of the regression method for influence coefficient estimation, only dependence of subsystem mass on GVM was considered [27]. For example, the linear additive model for body structure in these earlier applications was of the form,

$$\hat{m}_{BODY,STRUCTURE} = \beta_0' + \beta_1' (GVM) + \varepsilon'$$
model 1



This model has a serious flaw when there are other unaccounted-for mass drivers which are also correlated with GVM. For the body structure two significant mass drivers are gross vehicle mass and plan view area, as described earlier. The correlation coefficient between plan view area and gross vehicle mass is 0.80. Physically this describes the well known fact that cars which are larger (large plan view area) are also generally heavier (large GVM). For model 1, the coefficient β_l '=0.1732 reflects not only the dependence of body structure mass on GVM, but also the confounded dependence on plan view area. This artificially inflates the dependence on GVM, and thereby indicates an incorrectly high mass influence coefficient.

A more apt model for this case is

$$\hat{m}_{BODY,STRUCTURE} = \beta_0 + \beta_1(GVM) + \beta_2(PlanViewArea) + \varepsilon \qquad \text{model } 2$$

For model 2, the coefficient $\beta_1 = 0.1267$ is lower compared with $\beta_1' = 0.1732$ for model 1. This reduction is because the variability of body mass with plan view area is now being included in coefficient β_2 .

In the work described in this paper, an emphasis was placed on identifying the most important mass drivers for a more accurate estimate of influence coefficient. For example, the model for body structure mass is

 $\hat{m}_{BODY,STRUCTURE}$, $kg = -22.688 + 0.1267(GVM, kg) + 14.683(PlanViewArea, m^2) + \varepsilon(0, 21.8)$

This results in an estimate for the body structure mass influence coefficient of $\gamma_{\text{BODY.STRUCTURE}} = 0.1267 \pm 0.0257$

Table 1 is a summary of results for subsystem mass influence coefficients using the regression method.

| Subsystem | Mass drivers determined by statistical significance | Regression based mass influence coefficient |
|------------------|--|--|
| Body Structure | • Gross vehicle mass • Plan view area | 0.127 ± 0.033 |
| Powertrain | (Overall length x Overall width) • Gross vehicle mass • Acceleration (0-100 km/h time) | 0.117 ± 0.034 |
| Front Suspension | • Gross front axle mass (~0.6 x Gross vehicle mass for front wheel drive) | Both front and rear suspensions |
| Rear Suspension | • Gross rear axle mass (~0.4 x Gross vehicle mass for front wheel drive) | 0.055 ± 0.012 |
| Steering System | • Gross front axle mass (~0.6 x Gross vehicle mass for front wheel drive) | 0.009 ± 0.003 |
| Braking System | • Gross vehicle mass | 0.024 ± 0.007 |
| Tires & Rims | • Gross vehicle mass | 0.050 ± 0.012 |
| Fuel Tank System | • Gross vehicle mass, engine displacement | $\begin{array}{llllllllllllllllllllllllllllllllllll$ |

Table 1Subsystem mass drivers and influence coefficients



Finally, it must be noted that while a regression model provides an estimate of influence coefficient, it does not provide physical insight into the nature of the dependency between subsystem mass and mass drivers. In this sense it is inferior to the analytical approach which offers this insight. A further shortcoming of the regression method is that when a significant mass driver which covaries with GVM is overlooked, the influence coefficient will be inflated.

7. Comparison of Influence Coefficients by Analytical and Regression Methods—A summary of the results for the estimated mass influence coefficients is shown in Table 2 and Figure 7. Agreement between the two methods is good, with the analytical method generally predicting values slightly lower than the regression method. Of particular interest is the vehicle influence coefficient—the sum of the subsystem coefficients where agreement is within 20 %, left side of Figure 8.

| | Mass influence coefficient | | | |
|------------------|----------------------------|-------------------|--|--|
| Subsystem | Analytical (fka) | Regression (UofM) | | |
| Body Structure | 0.096 | 0.127 ±0.026 | | |
| Suspension | 0.050 | 0.055 ± 0.012 | | |
| Braking System | 0.037 | 0.024 ± 0.007 | | |
| Powertrain | 0.106 | 0.117 ± 0.034 | | |
| Fuel Tank System | 0.010 | 0.026 ±0.011 | | |
| Steering System | 0.007 | 0.009 ± 0.003 | | |
| Tires and Rims | 0.036 | 0.050 ± 0.012 | | |
| Sum | 0.342 | 0.406 ± 0.052 | | |
| | Table 2 | | | |

Summary of subsystem mass influence coefficients



Figure 7 Comparison of subsystem influence coefficients by analytical and regression methods





Figure 8 Comparison of vehicle influence coefficients by analytical and regression methods

8. Secondary Mass Change Model Application in Vehicle Design—A primary application of secondary mass change modeling is to aid in setting a mass budget for a new vehicle design. This budget sets a curb and gross vehicle mass used to size the subsystems during design. It also sets a mass target for each subsystem to be met; in this way it is similar to a cost budget. Setting the mass budget too low will result in an unachievable mass goal, resulting in detrimental mass increases as the design progresses. On the other hand, setting the mass budget too high will result in an inefficient and uncompetitive vehicle.

The challenge in setting a mass budget is to ensure the vehicle mass will be consistent with fuel consumption and acceleration requirements for the vehicle. Inevitably mass reduction compared to the current vehicle will be required, and the mass budget will ensure that this reduction is done cost effectively. Figure 9 illustrates a sequence of seven steps to set an efficient mass budget (this process uses secondary mass change modeling in Steps 4 and 6).





Sequence of seven steps to set an efficient mass budget

Step 1—A target mass for the vehicle system is set which is consistent with fuel consumption and acceleration requirements. Frequently this target mass is a result of powertrain modeling and competitive benchmarking.

Step 2—A reference vehicle is identified with known subsystem mass and curb mass. Some criteria for selection of the *reference vehicle* include having proven contemporary technologies, competitive in the market place, approximately the size and subsystem content as the vehicle under design. Often the prior model of the vehicle under design becomes the *reference vehicle*. This *reference vehicle* will be used in step 5 to size specific mass reduction technologies for the vehicle under design.

Step 3—The required light weighting is identified as the difference of the *reference vehicle* curb mass and the target curb mass set in step 1.

Step 4—The required light weighting is separated into a primary mass component, Δ , and secondary mass component, $\Delta \Gamma$. The primary mass reduction is the sum of several light weighting technologies selected to be applied to the new vehicle program (a method for this is discussed in step 5).

(Required light weighting) =
$$\Delta + \Delta\Gamma$$

$$\Delta = \frac{(\text{Required light weighting})}{(1+\Gamma)}$$
equation 71

The type of secondary mass coefficient used depends on the time horizon of the vehicle program: a) For short-term with all carry over subsystems. use no secondary mass change, b) for mid-term



enhancement, use simple secondary mass change, c) for long-term platform redesign, use compounded secondary mass change.

Step 5—A set of mass reduction technologies must be identified which will yield the required total primary mass reduction.

As an example, consider a new vehicle program with the following requirements and constraints;

- Primary mass reduction required: 50.0 kg
- Investment constraint <\$500,000
- Piece cost constraint <\$60.00

Table 3 shows seven example mass reducing technologies under consideration. The mass savings for each technology relative to the *reference vehicle* has been estimated (column 2). The *reference vehicle* mass has also been used to size the subsystem to arrive at the possible mass reduction. To ensure that the technologies are adopted in the most efficient order, the marginal cost—piece cost per unit mass saved—is calculated (column 6). Piece cost increase and investment required to implement the technology are also shown (columns 3 and 4). The technologies are now ordered based on increasing marginal cost. By adopting the technologies in this order until the required total primary mass reduction is met will ensure the most efficient set.

| | Based on | | | | | Calculate | ed value. | s |
|----------------------|------------------------------------|---------|-------------|-------------|------------------|-----------|-----------|------------|
| | sizing for Reference Vehicle | | | | Marginal Cost | | -Cumula | tive |
| Mass reduction | | | | | | | | |
| technology | Mass | | | | | Mass | Piece | |
| sorted by increasing | savings | Piece | Investment | | Cost/unit | saved | cost | Investment |
| marginal cost | (kg) | cost \$ | required \$ | Subsystem | mass \$/kg | kg | \$ | \$ |
| | | | | | 0 | 0 | 0 | 0 |
| Minimum capacity | | | | | | | | |
| wheels and tires | 10 | \$0 | \$10,000 | Tire& wheel | \$0.00 | 10 | \$0 | \$10,000 |
| Reduce glass | | | | Non | | | | |
| thickness | 10 | \$3 | \$10,000 | structure | \$0.30 | 20 | \$3 | \$20,000 |
| Rear suspension | | | | Rear | | | | |
| optimization | 10 | \$5 | \$200,000 | suspension | \$0.50 | 30 | \$8 | \$220,000 |
| Sound treatment | | | | Non | | | | |
| optimization | 20 | \$50 | \$15,000 | structure | \$2.50 | 50 | \$58 | \$235,000 |
| Body joint | | | | Body | | | | |
| improvements | 15 | \$50 | \$500,000 | structure | \$3.33 | 65 | \$108 | \$735,000 |
| Tubular brake pedal | | | | | | | | |
| bracket | 5 | \$40 | \$50,000 | Braking | \$8.00 | 70 | \$148 | \$785,000 |
| Side door material | | | | 0 | | | | |
| change | 10 | \$100 | \$100,000 | Closures | \$10.00 | 80 | \$248 | \$885,000 |

Table 3

Mass reduction technologies sorted by marginal cost (Step 5 of Figure 9)

The two graphs of Figure 10 show the cumulative mass reduction vs. cumulative piece cost, Figure 10(a), and cumulative mass reduction vs. cumulative investment, Figure 10(b). The required primary mass reduction is the green zone in upper left of each graph, and the red zone, right side of each graph, indicates the region prohibited by the constraints on piece cost and investment respectively.



For this example, selecting first four technologies will achieve the required 50 kg primary mass reduction with a total piece cost of \$58 and total investment of \$235,000; meeting both constraints. The marginal cost for entry of additional technologies if needed during later design is then \$2.50/kg, that of the last technology accepted.



Step 6—Apply secondary mass reduction to each subsystems of the *reference vehicle* using the primary mass reduction determined in Step 5. The model is equation 5 rewritten here

$$mi_{RS} = mi_0 + \Delta_i + \Delta \tau$$
 equation 5 repeated

The value for τ , the secondary mass coefficient, is that used in step 4. The subsystem mass, mi_0 , is from step 2. The technology enabled primary mass change, Δ_i , is from step 5 shown in Table 3 columns 5 and 2.

Step 7—The subsystem masses that result from step 6 will sum to the target vehicle mass determined in step 1. In this way the mass budget is based on the 'real' measured mass of the *reference vehicle* with subsystem masses scaled to function at the target vehicle mass.

This seven step process provides a framework for budgeting mass at the start of a vehicle design program. It is also of interest to researchers studying alternative subsystem technologies and materials and seeking to understand the vehicle mass implications. As it mimics the design process, secondary mass change effects from technology or material substitution can be evaluated using this process. The type of secondary mass coefficient used depends on the time horizon of the vehicle program: a) For short-term with all carry over subsystems, use no secondary mass change, b) for mid-term enhancement use simple secondary mass change, c) for long-term platform redesign use compounded secondary mass change. It is important to also remember that the *reference vehicle* parameters are used to size the subsystem technologies being evaluated.



9. Conclusions and Observations—

1) The Analytical and Regression methods are complimentary in providing upper and lower bounds on the true value of influence coefficient. The Regression method approaches the true influence coefficient from above. This is because any unaccounted for mass drivers which are correlated with vehicle mass will inflate the estimated influence coefficient. The Analytical method approaches the true influence coefficient from below. This is because there are other mass-dependent requirements which were not accounted for and thus the model will under estimate the vehicle mass dependence of the subsystem.

2) Often in the literature, only the vehicle secondary mass coefficient, Γ , is provided without specifying if the value is for simple secondary mass or compounded secondary mass change, for example "*the secondary mass coefficient is 0.30*". This has caused undue problems in comparing coefficients across research papers. It is suggested that when a number is used, it is specified as simple or compounded.

3) The term mass *Decompounding* has entered the lexicon of secondary mass analysis. It is the authors' opinion that this term should be discontinued as it only adds to confusion in a rapidly developing field. The origin is probably due to a desire to cast secondary mass change in a positive light—as a reduction rather than an increase. However, this term does not aptly describe the secondary mass change process. We are not taking apart a system, as the term implies. In all cases we are compounding mass; a positive mass for an increase, a negative mass for a reduction; the same process is occurring in either case. So the addition of another term erroneously implies two processes; one for increasing mass—*compounding*, and one for decreasing mass—*decompounding*. There is a very direct analogy with monetary interest terms: Simple interest and Compounded interest; no need for extra terms. Appendix 1 contains a list of suggested terms for secondary mass change modeling.

4) For estimates of changes in fuel consumption or use stage greenhouse gas, understanding how *vehicle* mass changes is sufficient. For those applications, knowledge of the vehicle secondary mass coefficient, γ_V , is sufficient. However, for estimates of material production stage greenhouse gas, the vehicle coefficient is not sufficient. This is because the change in mass for each specific material is needed. Material mass is linked to the material content of individual subsystems. Therefore the individual subsystem influence coefficients, γ_i , must be known to predict changes in subsystem mass and their constituent materials. See Appendix 2 for a means to calculate material mass from subsystem mass using a vehicle Bill of Materials.



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Appendix 1

Terminology for Secondary Mass Change Modeling

Bill of Materials - A matrix relating material content fraction for each subsystem (Appendix 2).

- Compounded secondary mass change -The change in vehicle mass after a series of subsystem resizings (kg).
- Compounded secondary mass coefficient -The change in vehicle mass after a series of subsystem resizings per unit of primary mass change (kg/kg).
- Curb mass Mass of the vehicle with fluids, without occupants or cargo (kg).
- GVM Gross vehicle mass, the design mass for the fully ladened vehicle (kg).
- *Primary mass change* An initial mass change in a component (positive value is a mass increase, negative a mass reduction) (kg).
- *Reference Vehicle* The vehicle for which subsystems are initially sized prior to any mass changes.
- Secondary mass change The mass change due to subsystem resizing (kg). May be either simple or compounded.
- Simple secondary mass change The change in vehicle mass after resizing subsystems only one time (kg).
- Simple secondary mass coefficient The change in vehicle mass after resizing subsystems once per unit of primary mass change (kg/kg).
- Subsystem The set of components which perform a specific function in the vehicle system. Examples: 1) Powertrain subsystem with components: engine, transmission, drive shafts, engine cooling, starter and battery. Function: Propel vehicle. 2) Body structure subsystem having components: body shell, engine cradle, suspension cradles. Function: React service, crashworthiness, and stiffness load conditions.
- Subsystem influence coefficient Change in subsystem mass per unit change in gross vehicle mass (kg/kg).

Vehicle influence coefficient - Sum of subsystem mass influence coefficients (kg/kg).



Appendix 2

Vehicle Bill of Materials and Material Mass

To capture material use in the vehicle, the Bill of Materials—BOM—is used. By using the BOM, secondary mass changes for subsystems may be used to estimate changes in mass for a particular material. The BOM is a matrix with each element being the fraction of a particular subsystem composed of a particular material,

٦

BOM
$$\overline{\alpha} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots \\ \alpha_{21} & \alpha_{ij} & \dots \\ \dots & \dots & \dots \end{bmatrix}$$
 equation A1

where α_{ij} = fraction of subsystem i composed of material j (Note that rows will sum to 1)



Figure A1

Using the secondary mass change models described earlier, a subsystem mass vector may be found

$$\overline{m} = \begin{bmatrix} m_1 \\ m_i \\ \dots \end{bmatrix}$$
 equation A2

where m_i *=mass of subsystem i*



The total mass of material j in vehicle is then

$$\overline{\mathbf{m}} = \begin{bmatrix} \mathbf{m}_1 & \mathbf{m}_j & \dots \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 & \mathbf{m}_i & \dots \end{bmatrix} \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots \\ \alpha_{21} & \alpha_{ij} & \dots \\ \dots & \dots & \dots \end{bmatrix} = (\overline{\mathbf{m}})^T (\overline{\alpha}) \qquad \text{equation A3}$$

where $m_j = total mass of material j in vehicle$

Appendix 3

Formula Symbols and Indices (Analytical Method, Section 5)

Powertrain

| ρ_{Stahl} | [kg/m ³] | density of steel |
|-----------------------------|----------------------|--|
| $	au_{b,W}$ | [N/mm ²] | torsion fatigue strength |
| ϕ_1 | [-] | progressive ratio of gearbox |
| ϕ_2 | [-] | progressive factor of gearbox |
| $\sigma_{H,lim}$ | $[N/mm^2]$ | fatigue strength of material |
| a _{Get} | [mm] | distance of gearbox input to gearbox output shaft |
| A_L | [-] | number of bearings of gearbox shafts |
| b/d_1 | [-] | relationship gear wheel width to diameter (1 st gear) |
| b _L | [mm] | width of bearings of gearbox shafts |
| b _{r,i} | [mm] | width gear pinions of gearbox |
| b _{Sk} | [mm] | width detents of gearbox |
| B _s | [-] | number detents of gearbox |
| d_{AW} | [mm] | diameter of drive shaft |
| d _{Get} | [mm] | diameter of gearbox |
| $d_{\text{GEW},\text{min}}$ | [mm] | diameter of gearbox input shaft |
| d _{KW} | [mm] | diameter of cardan shaft |
| d _{r,n} | [mm] | diameter gear pinions of gearbox |
| $d_{z,n}$ | [mm] | diameter gear wheels of gearbox |
| F _B | [N] | acceleration resistance |
| F_{Bed} | [N] | driving resistance |
| F_{L} | [N] | air resistance |
| F _R | [N] | rolling resistance |
| F _{St} | [N] | grade resistance |
| G _{AEW} | [kg] | weight of engine |
| G _{AW} | [kg] | weight of drive shaft |
| G _{DZW,KDP} | [kg] | weight of clutch pressure plate |
| G _{DZW,KS} | [kg] | weight of clutch disc |
| G _{DZW,SR} | [kg] | weight of fly wheel of engine |
| G _{F,KW} | [kg] | weight of cooling water |
| G _{F,MÖ} | [kg] | weight of engine oil |
| G _K | [kg] | weight of cooling system |



| G _{KW} | [kg] | weight of cardan shaft |
|---------------------|--------------------|--|
| G _{r,i} | [kg] | weight gear pinion of gearbox |
| G _{ZES,OM} | [kg] | weight of 12 volt battery for gasoline engines |
| G _{ZES,DM} | [kg] | weight of 12 volt battery for diesel engines |
| i _{Diff.} | [-] | differential ratio |
| i _{G,ges} | [-] | gearbox ratio spread |
| i _n | [-] | gearbox ratio |
| K _A | [-] | application factor of gear wheels |
| K _{Hα} | [-] | front factor of gear wheels |
| K _{Hβ} | [-] | width factor shoulder pressing of gear wheels |
| Kv | [-] | dynamic factor of gear wheels |
| l _{Get} | [mm] | length of gearbox |
| l_{AW} | [mm] | length of drive shaft |
| l _{KW} | [mm] | length of cardan shaft |
| M _{Antr.} | [Nm] | engine torque |
| n _{Antr.} | [U/min] | engine speed |
| P _{Antr.} | [kW] | driving power |
| P _{Bed} | [kW] | power demand |
| r _{dyn} | [m] | dynamic wheel radius |
| S _{AW} | [-] | safety index of drive shafts |
| S _H | [-] | safety index of distance gearbox shafts |
| S _{KW} | [-] | safety index of cardan shafts |
| t _{KW} | [mm] | thickness of cardan shaft |
| V _{Fzg} | [km/h] | vehicle velocity |
| Z | [-] | number of gears (gearbox) |
| Z _β | [-] | skew factor of gear wheels |
| Z _{B/D} | [-] | meshing factor of gear wheels |
| Z _E | $[(N/mm^2)^{0,5}]$ | elasticity factor of gear wheels |
| Zε | [-] | contact ratio factor of gear wheels |
| Z _H | [-] | spread factor of gear wheels |
| Z _L | [-] | lubricant factor of gear wheels |
| Z _{NT} | [-] | durability factor of gear wheels |
| Z _R | [-] | surface quality of gear wheels |
| Z _V | [-] | velocity factor of gear wheels |
| Z _W | [-] | material pairing factor of gear wheels |
| Z _X | [-] | size factor for shoulder pressing of gear wheels |
| | | |

Fuel Tank System

| Δv_{NEDC} | [km/h] | speed difference NEDC |
|--------------------------|------------|--|
| a _V | [-] | coefficient 1 of zero-power consumption |
| b _V | [-] | coefficient 2 of zero-power consumption |
| c _V | [-] | coefficient 3 of zero-power consumption |
| G _{KT} | [kg] | weight of fuel tank |
| Pe | [kW] | effective power |
| S _{Ref.} | [km] | range of reference vehicle |
| v_{1000} | [km/h] | related vehicle velocity at 1000 U/min |
| V _{Fzg} | [km/h] | vehicle velocity |
| Vgew.red. | [l/100 km] | fuel consumption of primary weight reduced vehicle |



| $V_{\rm H}$ | [1] | engine displacement |
|--------------------------|------------|--|
| V _{KT} | [1] | fuel tank volume |
| V _{KT,gew.red.} | [1] | fuel tank volume of primary weight reduced vehicle |
| V _{KT,Ref.} | [1] | fuel tank volume of reference vehicle |
| V _{Null} | [l/h] | zero-power fuel consumption |
| V _{Pe} | [l/h] | effective power fuel consumption |
| V _{Ref.} | [l/100 km] | fuel consumption of reference vehicle |
| Z _{Pe} | [l/kWh] | coefficient effective power fuel consumption |

Suspension

| G _{Fzg.,zul.} | [kg] | gross vehicle mass |
|------------------------|------|--|
| G _{VD,VA} | [kg] | weight vertical dynamics of front axle |
| G _{VD,HA} | [kg] | weight vertical dynamics of rear axle |

Braking System

| | r 1 | |
|-----------------------------------|--------------|--|
| $\mu_{\text{Str.}}$ | [-] | static friction coefficient of the street |
| a _{Br} | $[m/s^2]$ | average braking deceleration |
| D _{Br,S,außen,v,i} | [mm] | outer diameter of braking disc front |
| D _{Br,S,außen,h,i} | [mm] | outer diameter of braking disc rear |
| D _{Br,S,h} | [mm] | effective diameter of braking disc rear |
| D _{Br,S,h,gew.red} | [mm] | effective diameter of braking disc rear of prim. weight reduced vehicle |
| $D_{Br,S,v}$ | [mm] | effective diameter of braking disc front |
| D _{Br,S,v,gew.red} | [mm] | effective diameter of braking disc front of prim. weight reduced vehicle |
| F _{Br,S,v} | [N] | braking power at an effective diameter of front wheel |
| F _{Br,S,h} | [N] | braking power at an effective diameter of rear wheel |
| F _{Br,v} | [N] | maximum braking power of front wheel |
| F _{Br,v,gew.red} . | [N] | maximum braking power of front wheel of prim. weight reduced vehicle |
| F _{Br,h} | [N] | maximum braking power of rear wheel |
| F _{Br,h,gew.red} . | [N] | maximum braking power of rear wheel of prim. weight reduced vehicle |
| F _{RZh} | [N] | dynamic wheel load of rear axle |
| F _{R,Z,v} | [N] | dynamic wheel load of front axle |
| g | $[m/s^2]$ | gravity acceleration |
| G _{Fzg.,zul.} | [kg] | gross vehicle mass |
| G _{FW.,Br,S,innenbel.,v} | [kg] | weight of inner ventilated braking disc front |
| G _{FW.,Br,S,innenbel.,h} | [kg] | weight of inner ventilated braking disc rear |
| G _{FW.,Br,S,massiv,v} | [kg] | weight of massive braking disc front |
| G _{FW.,Br,S,massiv,h} | [kg] | weight of massive braking disc rear |
| h | [mm] | height centre of gravity of overall vehicle |
| 1 | [mm] | wheel base of overall vehicle |
| l _h | [mm] | distance centre of gravity to rear axle |

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| l_v | [mm] | distance centre of gravity to front axle |
|----------------------|--------|---|
| r _{dyn} | [m] | dynamic wheel radius |
| S _{100km/h} | [m] | braking distance (vehicle velocity100 km/h) |
| V _{Fzg} | [km/h] | vehicle velocity |

Steering System

| G _{Fzg.,zul.} | [kg] | gross vehicle mass |
|------------------------|------|---------------------------|
| G _{LS} | [kg] | weight of steering system |

Tires and Rims

| ρ _{Stahl} | [kg/m ³] | density of steel |
|------------------------------------|----------------------|--|
| A _{Flanke,R,i} | [mm ²] | area of tire shoulder |
| A _{Flanke,R,Serie50} | [mm ²] | area of tire shoulder of radial tire series 50 |
| A _{Flanke,R,Serie50,16} ' | [mm ²] | area of tire shoulder of radial tire series 50 (16" diam.) |
| A _{Flanke,R,Serie55} | [mm ²] | area of tire shoulder of radial tire series 55 |
| A _{Flanke,R,Serie55,17} " | [mm ²] | area of tire shoulder of radial tire series 55 (17" diam.) |
| A _{Flanke,R,Serie60} | [mm ²] | area of tire shoulder of radial tire series 60 |
| A _{Flanke,R,Serie60,18} " | [mm ²] | area of tire shoulder of radial tire series 60 (18" diam.) |
| A _{Flanke,R,Serie65} | [mm ²] | area of tire shoulder of radial tire series 65 |
| A _{Flanke,R,Serie65,15} " | [mm ²] | area of tire shoulder of radial tire series 65 (15" diam.) |
| b _R | [mm] | tire width |
| $B_{M,F}$ | [mm] | rim width |
| D _{Felge} | [mm] | rim diameter |
| F _{Z,W,h} | [N] | maximum static wheel load of rear axle |
| F _{Z,W,max} | [N] | maximum static wheel load |
| $F_{Z,W,v}$ | [N] | maximum static wheel load of front axle |
| G _{b,Serie50,16} " | [kg] | weight of radial tire series 50 (16" diameter) |
| G _{b,Serie55,17} " | [kg] | weight of radial tire series 55 (17" diameter) |
| G _{b,Serie60,18} " | [kg] | weight of radial tire series 60 (18" diameter) |
| G _{b,Serie65,15} " | [kg] | weight of radial tire series 65 (15" diameter) |
| G _{Fzg.,zul.} | [kg] | gross vehicle mass |
| G _{FW,F} | [kg] | weight rim |
| G _{FW.,Reifen,Serie50} | [kg] | weight of radial tires series 50 |
| G _{FW.,Reifen,Serie55} | [kg] | weight of radial tires series 55 |
| G _{FW.,Reifen,Serie60} | [kg] | weight of radial tires series 60 |
| G _{FW.,Reifen,Serie65} | [kg] | weight of radial tires series 65 |
| G _{R,Z} | [kg] | tire load |
| k _{L/F} | [-] | relationship of shoulder height to tread of tire |
| 1 | [mm] | wheel base of overall vehicle |
| l _h | [mm] | distance centre of gravity to rear axle |
| l _v | [mm] | distance centre of gravity to front axle |
| S_R | [-] | safety index of tire dimensioning |
| t_{Felge} | [mm] | rim sheet thickness |



Appendix 4

Formula Symbols and Indices (Regression Method, Section 6)

| M_0 | [kg] | Initial vehicle mass for which the subsystems are sized |
|------------------------|---------|---|
| M_{RS} | [kg] | Vehicle mass after resizing subsystems |
| m_{i0} | [kg] | Initial subsystem i mass (From Reference Vehicle) |
| m_{iRS} | [kg] | Resized subsystem i mass |
| \hat{m}_i | [kg] | Estimated mass for subsystem i using regression |
| (mass driver)_ | | Value for a vehicle or subsystem attribute upon which subsystem mass depends |
| r _e | - | Multiplier which indicates the residual error for a power |
| | | regression model ($r_{\varepsilon}=1$ for an exact match of model with data) |
| β | | Coefficients estimated by regression |
| Δ | [kg] | Initial total mass change (primary mass change) |
| ΔΓ | [kg] | secondary mass change due to resizing subsystems |
| Δi | [kg] | Initial mass change in subsystem i |
| $\varDelta \cdot \tau$ | [kg] | Additional (secondary) mass change for subsystem i |
| ε | [kg] | Residual error for a linear regression model of subsystem mass |
| γ_V | [kg/kg] | Mass influence coefficient for the vehicle given by |
| | | $\gamma_V = \Sigma \gamma_i$ |
| γi | [kg/kg] | Mass influence coefficient for subsystem i |
| τ | [kg/kg] | Secondary mass coefficient for subsystem which |
| | | depends on the treatment of resizing iterations: Simple |
| | | for one resizing, <i>Compounded</i> for an infinite series of resizings. |
| Г | [kg/kg] | Secondary mass coefficient for the vehicle which |
| | | for one resizing, <i>Compounded</i> for an infinite series of |
| | | resizings. |



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